Exercise 39

For the following exercises, find $(f \circ g)$ and the domain for $(f \circ g)(x)$ for each pair of functions.

$$f(x) = \frac{x+1}{x+4}, \quad g(x) = \frac{1}{x}$$

Solution

Calculate $(f \circ g)(x)$ by plugging the formula for $g(x)$ in where x is in the formula for $f(x)$.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= \frac{\frac{1}{x}+1}{\frac{1}{x}+4} \\ &= \frac{\frac{1}{x}+1}{\frac{1}{x}+4} \times \frac{x}{x} \\ &= \frac{1+x}{1+4x}\end{aligned}$$

The denominator of this rational function cannot be zero at any step.

$$x \neq 0 \quad \text{and} \quad 1+4x \neq 0$$

Solve for x.

$$x \neq 0 \quad \text{and} \quad x \neq -\frac{1}{4}$$

Therefore, the domain of $(f \circ g)(x)$ is

$$\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, 0\right) \cup (0, \infty).$$